APPENDIX B

THERMODYNAMIC ANALYSIS

Topics covered are selected materials from thermodynamics. Included are areas which are the most likely to be less familiar to a general auditor.

Psychrometrics

Psychrometrics is the study of moist air equilibrium thermodynamic processes. Why is it important? People need to maintain an internal environment that is comfortable (temperature, humidity, fresh air). Therefore, the HVAC system must regulate all three variables.

VARIABLE	SUMMER	WINTER
Temperature	High	Low
Humidity	High	Low
Air Flow	Low	Low

The brief summary covers:

1. Properties of real air
2. Limitations due to saturation (Boiling Curve)
3. Definitions of state variable

- Humidity Ratio (lb of moisture/lb of dry air)
- Enthalpy (Btu/lb of dry air)
- Specific Volume = 1/Density

The molecular weight of air is given as:

$$m = 28.9645 \frac{lb_m}{lb \times mol}$$

Thus, the gas constant can be found for air, R_a, by dividing the universal gas constant by the molecular weight.

$$R_a = \frac{1545.32}{28.9645} = 53.352 \frac{ft \times lb}{lb_m \times R}$$

Properties of Air

COMPONENT	% BY VOLUME
N_2	78.08
O_2	20.95
Ar	0.93
CO_2	0.03
Ne	0.0018
He	0.0005
CH_4	0.00015
H_2	0.00005
SO_2	Small
Kr	Small
Xe	Small
O_3	Small

Table 12.1: Dry Air Composition

Water Vapor

By manipulating the ideal gas equation, a relationship between the ideal gas law and the density for air can be developed.

$$PV = mRT \text{ or } \rho = m/V = P/RT$$

Looking at the new equation one can see that the density is inversely proportional to the to the gas constant R. So using the information obtained for air in the previous section the density of air to the density of water vapor based on proportionality can be compared. From this, it can be concluded that water vapor is much less dense than dry air.

$$r_a \propto \frac{1}{53.352} >> r_w \propto \frac{1}{85.778}$$

Real Air (Moist Air)

Realistically, air is not completely dry; it contains some moisture.

- x% Water Vapor
- (1-x)% Dry Air

In order to determine the density of real air, one must consider the densities of both dry air and water vapor.

$$\rho = \rho_a + \rho_w$$

Then substitute the densities with the ideal gas relation found in the previous section.

$$\begin{aligned} r &= \frac{P_a}{R_a T} + \frac{P_w}{R_w T} \\ &= \frac{P - P_w}{R_a T} + \frac{P_w}{R_w T} \\ &= \frac{P}{R_a T} - \frac{P_w}{R_a T}\left(1 - \frac{R_a}{R_w}\right) \\ &= \frac{P}{R_a T} - 0.378\frac{P_w}{R_a T} \end{aligned}$$

Amount of Water Vapor in a Moist Air Mixture

The amount of moisture in an air mixture is described by the humidity ratio, W. The humidity ratio can be defined by:

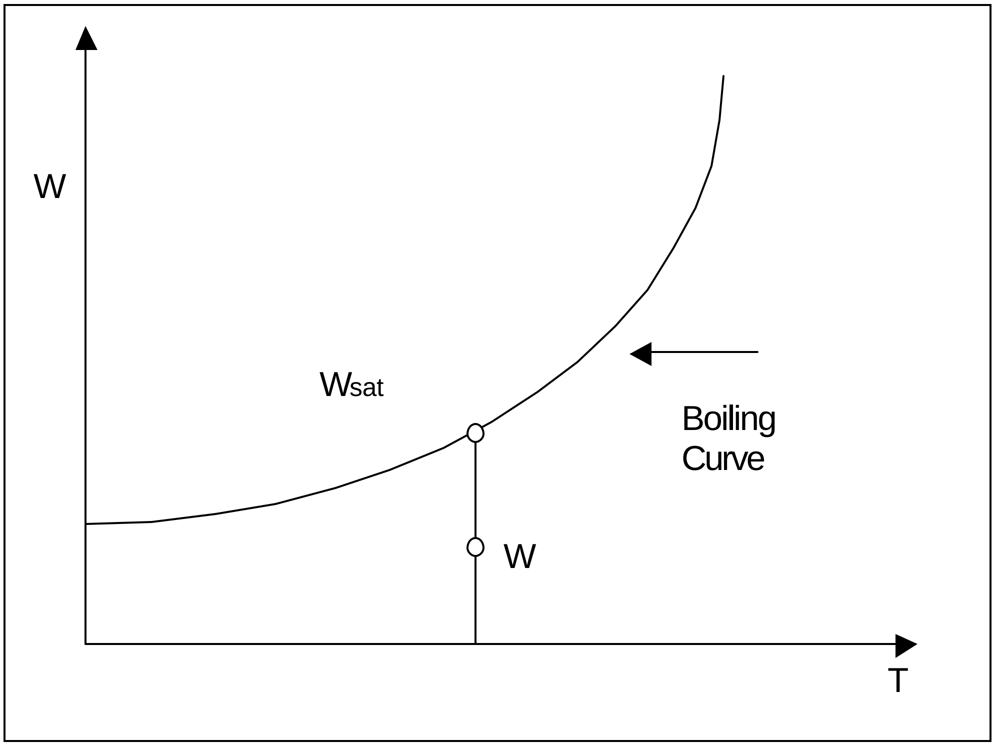
$$W = (\text{lb}_m \text{ of moisture } /V)/(\text{lb}_m \text{ of dry air } /V)$$

Some manipulation and substitution yields an expression for the humidity ratio.

$$W = \frac{x_w m_w}{x_a m_a} = \frac{\frac{m_w}{m_a} x_w}{x_a} = \frac{\frac{18.015}{28.9645} x_w}{1 - x_w}$$

$$W = \frac{0.622\frac{P_w}{P}}{1 - \frac{P_w}{P}} = 0.622\frac{P_w}{P - P_w} = 0.622\frac{P_w}{P_a}$$

This expression shows that the humidity ratio is proportional to the ratio of water pressure to the air pressure. The figure below shows how the humidity ratio varies with respect to temperature. As one can see, the humidity ratio increases significantly as temperature increases.



Energy Content

Enthalpy, h, is a measure of the energy content in the air. The enthalpy of an air/moisture mixture can be expressed as:

$$h = h_a + Wh_w$$

using

$$h_a = 0.24T$$
$$h_w = h_{fg}\text{ (at 32°F)} + C_{p,s}\,(T - 32)$$

where

h_{fg} = latent heat of vaporization, Btu/lb
$C_{p,s}$ = specific heat of water vapor = 0.444 Btu/lb-°F

Substituting these in for the first equation results in:

$h = 1075.15 + 0.444(T - 32)$

$\quad = 1061 + 0.444$

$\therefore\ h = 0.24T + W(1061 + 0.444T)$

where

T is in °F

W is in $lb_{m,w} / lb_{m,a}$

Relative Humidity

$$f = \frac{x_w}{x_{w,s}} = \frac{f\left(T, \frac{P_w}{P}\right)}{f(T)}$$

$$f = \frac{\frac{P_w}{P}}{\frac{P_{w,s}}{P}} = \frac{P_w}{P_{w,s}}$$

where $P_{w,s}$ is found from the Boiling Curve

$$0 \leq \phi \leq 1$$

Given ϕ and T, to get W:

1. $T \rightarrow P_{w,s}$ (from Boiling Curve)
2. $P_w = \phi P_{w,s}$
3. $W = 0.622\ (P_w / (P - P_w))$

Specific Volume

Specific volume is defined as the volume per unit mass.

$$v_a = \frac{v}{m_a}$$

Once again using the ideal gas law

$$PV = RT$$

$$v_a = \frac{R_a T}{P_a} = \frac{R_a T}{P - P_w} = \frac{R_a T}{P\left(1 - \frac{P_w}{P}\right)}$$

$$\frac{R_a T}{P} = 1 + 1.608W$$

Since specific volume is volume divided by mass, it can also be defined as the inverse of density (mass divided by volume).

$$r = \frac{1}{v_a}$$

§ *Psychrometric Example*

Given: $T_1 = 90°$ F, $\phi = 0.90$
Calculate the energy per pound of dry air to cool to 57°F, $\phi = 1$.

Method 1 (Analytical)
- At State 1:

$x_{w,1} = 0.90 = P_{w,1} / P_{ws,1}$
(From Table 2 in Chapter 6 of ASHRAE Fundamentals)
$P_{ws,1} = 0.6489$ psi

P = (0.90)(0.6489) = 0.629 psi
$W_1 = 0.622 \times 0.629(14.7 - 0.629) = 0.02780$ [lb_m / lb_a]
$h_1 = (0.24)(90) + (0.02780)[1061 + (0.444)(90)] = 52.2$

- State 3:

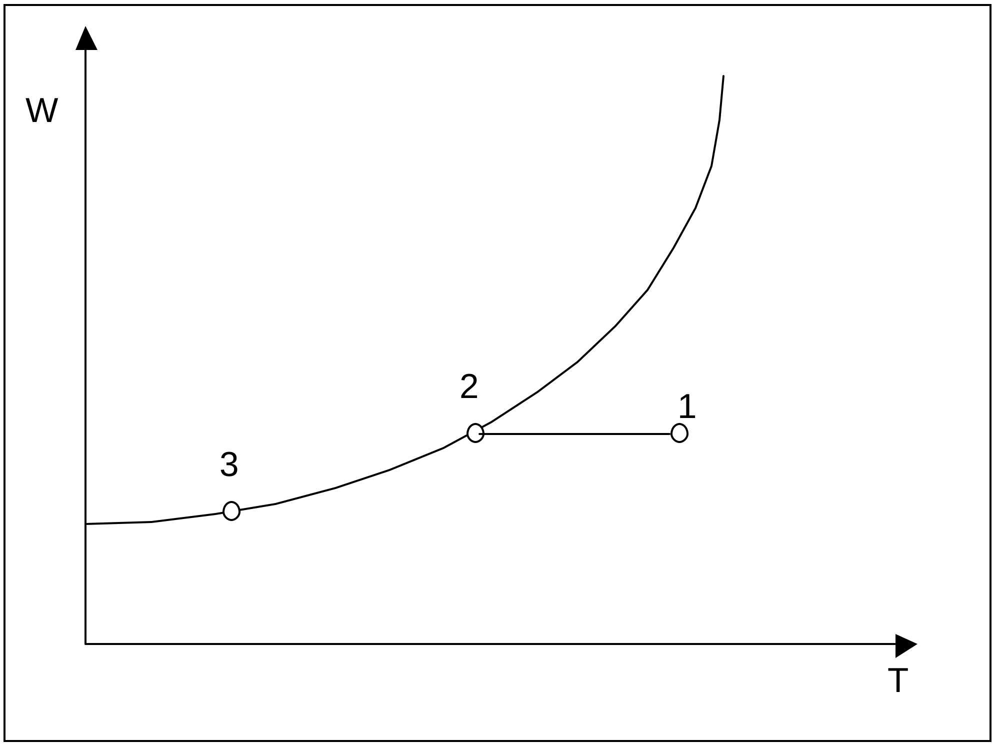
$P_{was,3} = P_{ws,57 \text{ deg } F} = 0.2302$ psi
h = (0.24)(57) + (0.009895)[1061 + (0.444)(57)] = 24.4 [Btu/lb]

$\therefore$ $\Delta h = 24.4 - 52.2 = -27.8$ [Btu/lb of dry air]

Method 2 (Graphical)
1. Locate point 1 at $T_1 = 90°$F, $\phi = 0.90$;
Read $h_1 \approx 52.5$ Btu/lb

2. Locate point 3 at T_3 = 57°F, ϕ = 1;
Read $h_3 \approx 24$ [Btu/lb]

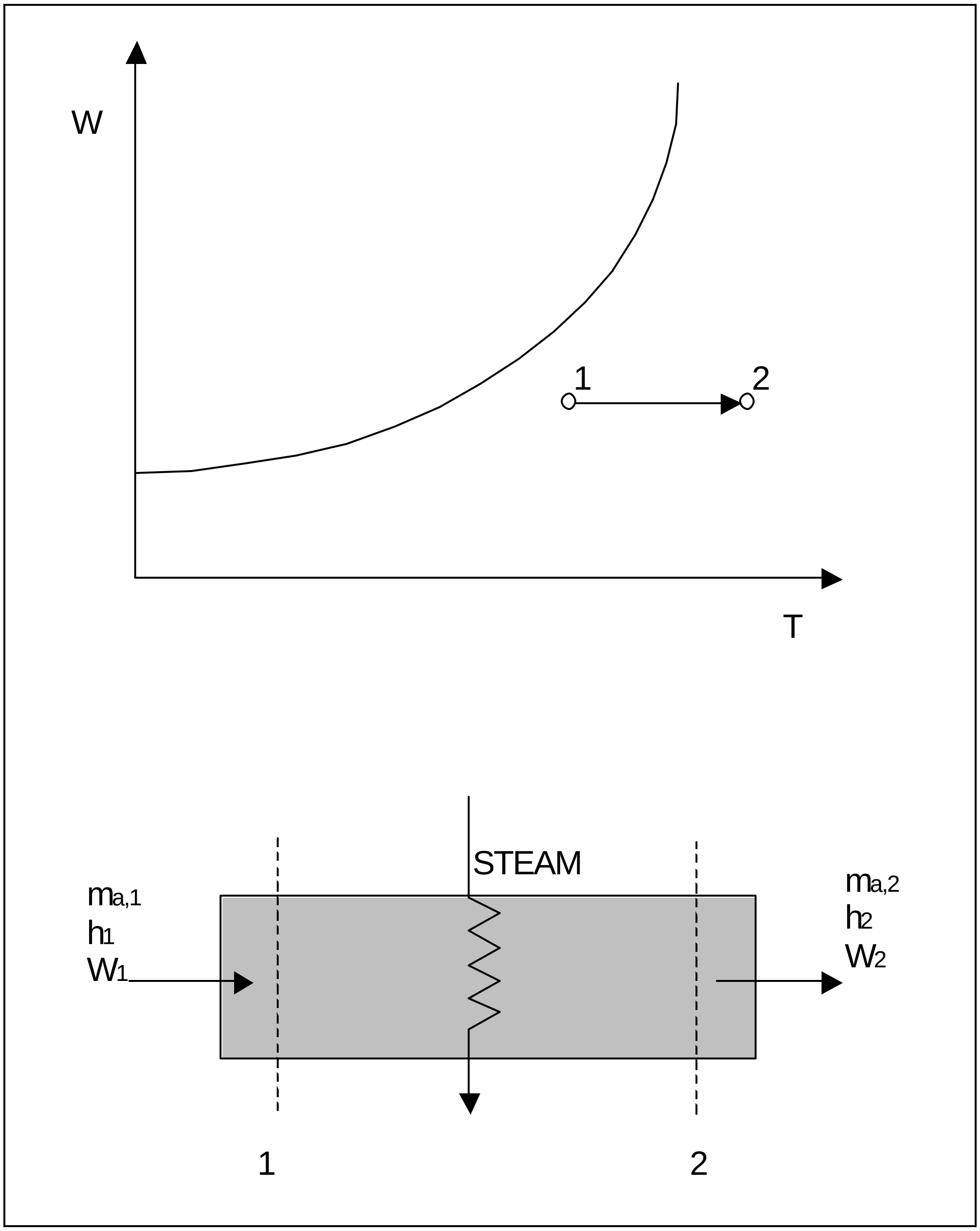
3. Calculate Δh
$\Delta h = 24 - 52.5 = -28.5$



Air Conditioning Processes

Air conditioning of air is done to ensure either proper conditions for a specific process or make more pleasant working environment for the people.

Heat Addition to Moist Air



Conservation of mass

$$\dot{m}_{a,1} = \dot{m}_{a,2}$$
$$\dot{m}_{a,1} W_1 = \dot{m}_{a,2} W_2 \rightarrow W_2 = W_1$$

Conservation of energy

$$q_{1\rightarrow 2} = \dot{m}_{a,1} \left(h_2 - h_1 \right)$$

§ *Example*

Given: T_1 = 35°F, ϕ_1 = 100%, 20,000 cfm_1
Air to be heated to 100°F
Find: The heater size required.

- State 1

Specific volume = 1/Density

$$v = \frac{1}{r} = \frac{RT}{P}[1 + 1.608W]$$

$$r = \frac{P_{a,1}}{R_a T(1 + 608W_1)}$$
$$= \frac{(14.7)(144)}{(53.35)(460 + 35)(1 + 1.608W_1)}$$

$P_{ws,1}$ = 0.09998 → from tables or charts at 35°F

∴ P = (0.09998)(1) = 0.09998

W_1 = 0.622 x 0.09998 / (14.7 - 0.09998) = 0.004259

∴ h 1 = (0.24)(35) + (0.004259)[1061 + (0.444)(35)]

h_1 = 12.985 Btu/lb

- State 2

$W_2 = W_1 = 0.4259$
$h_2 = (0.24)(100) + (0.004259)[1061 + (0.444)(100)]$
$h_2 = 28.708$ Btu/lb

Calculate the mass flow rate of air:

$$\dot{m}_a = \left(20{,}000 \tfrac{ft^3}{\min}\right)\left(60 \tfrac{\min}{hr}\right) r_{a,1}$$

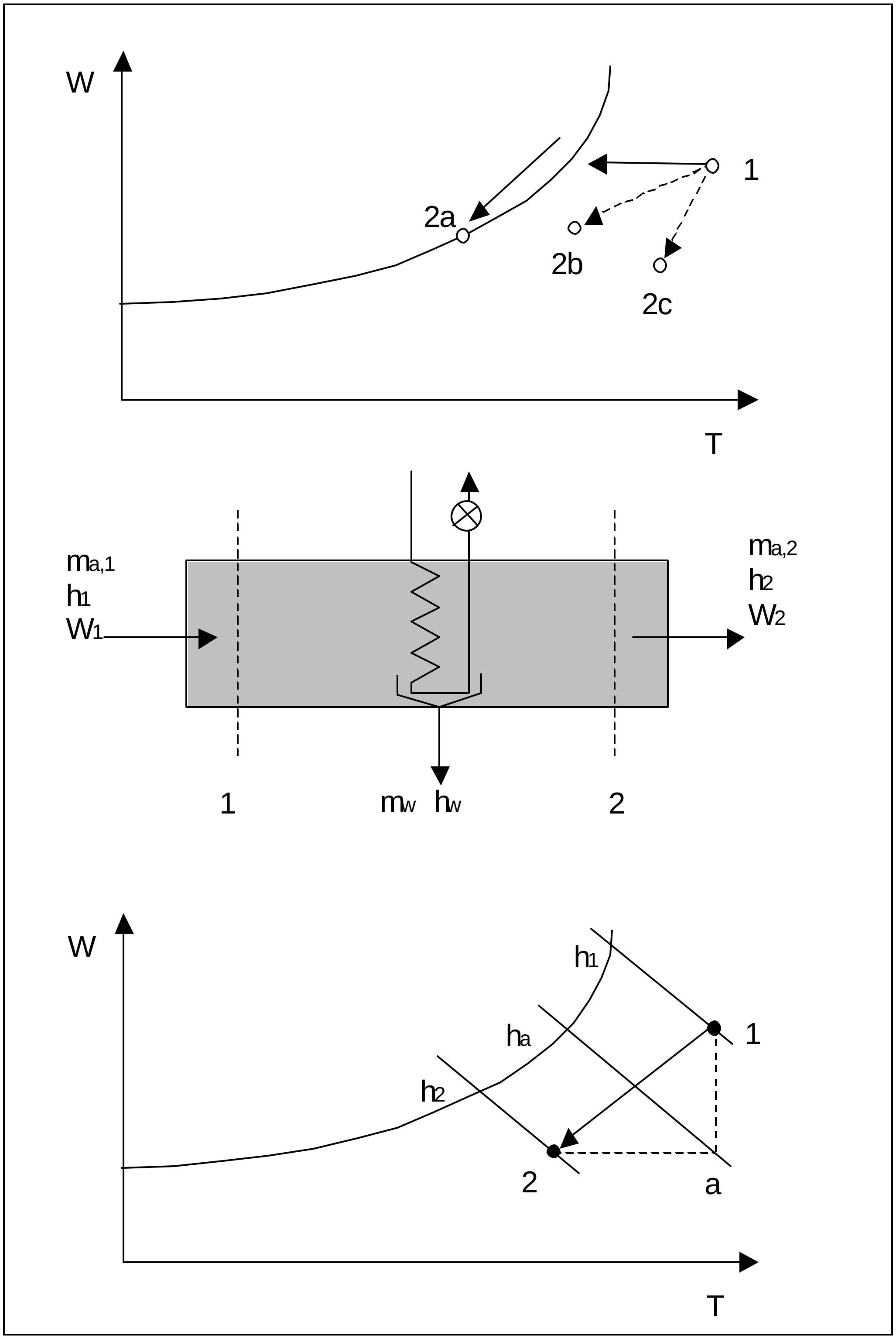
$$r_{a,1} = \frac{(14.7)(144)}{(53.35)(495)[1 + (1.608)(0.004259)]} = 0.07961 \frac{lb}{ft^3}$$

$$\dot{m}_a = \left(20{,}000 \tfrac{ft^3}{\min}\right)\left(60 \tfrac{\min}{hr}\right)\left(0.07961 \tfrac{lb}{ft^3}\right) = 95{,}534 \tfrac{lb}{hr}$$

$$q_{1\to 2} = \left(95{,}534 \frac{lb}{hr}\right)\left(28.708 - 12.985 \frac{Btu}{lb}\right) = 1.502 \frac{MMBtu}{hr}$$

$$q_{boiler} = \frac{1.502 \times 10^6}{h_{boiler}} = \frac{11.502 \times 10^6}{0.8} = 1.878 \times 10^6 \frac{Btu}{hr}$$

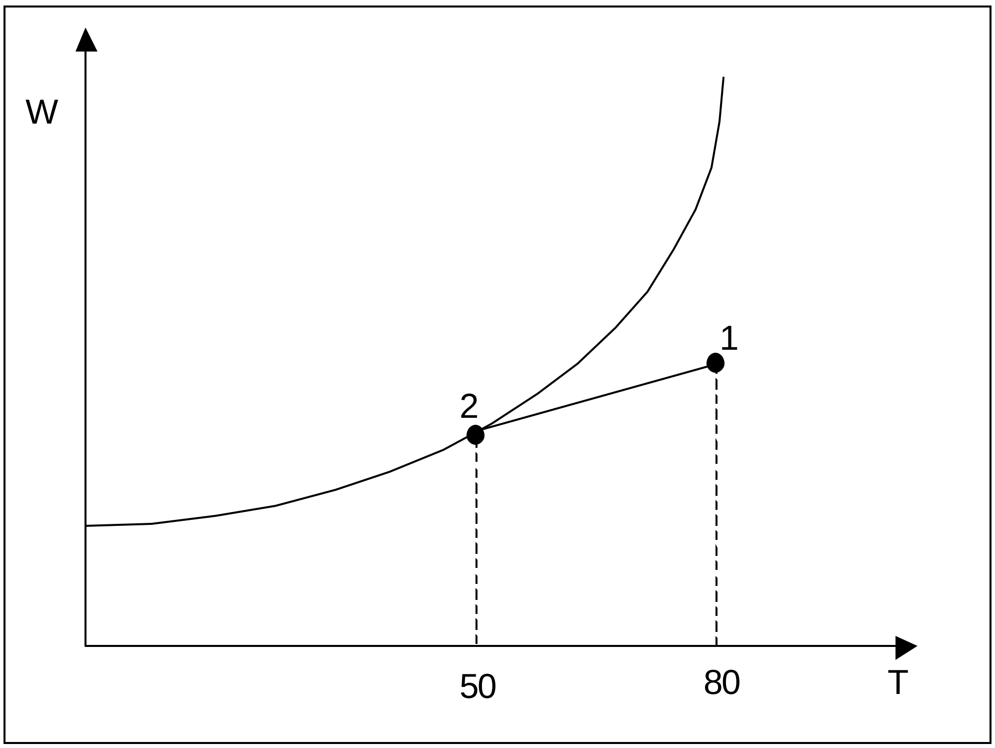
Cooling of Moist Air



$h_1 - h_a$ is all latent heat removal
$h_a - h_2$ is all sensible heat removal
$h_1 - h_2$ is total heat removal

§ *Example*

Determine the tons of refrigeration required to cool 10,000 cfm of air at 85°F dry bulb temperature, $\phi = 0.50$, to 50°F, $\phi = 1$



From the chart

$h_1 \approx 34.5$ $\quad v_1 \approx 14.01$
$h_2 \approx 20.2$ $\quad W_1 \approx 0.013$
$W_2 \approx 0.0076$

From tables, $h_{w,2} = 18.11$ Btu/lb$_a$
or
$h_{w,2} = C_p\,(T - 32) = 1\ \{\text{Btu/[lb deg F}(50 - 32 \text{ deg F})]\} = 18$ Btu/lb$_a$

$\dot{m}_a$ = (10,000 ft^3 /min) / (14.01 ft^3 /min) = 713.8 lb dry air per minute
$q_{1?2} = m_a\,[(h_2 - h_1) + (W_1 - W_2)h_w]$
= [713.8 (lb/min)] {[20.2 - 34.5 (Btu/lb)] + (0.013 - 0.0076)(18)(Btu/lb)}

= -10.138 Btu/min

That is, the heat removal rate is 10,138 Btu/min or 608,278 Btu/h.

$$1 \text{ ton of A/C} = (1 \text{ ton of ice/day}) \times (\text{day}/24\text{hr}) \times (144\text{Btu/lb}) \times (2{,}000 \text{ lb/ton})$$

where the latent heat of fusion for ice is 144 Btu/lb.

$$1 \text{ ton of A/C} = 12{,}000 \text{ Btu/h}$$

$$q = \frac{608{,}278 Btu/hr}{12{,}000 Btu/hrton} = 50.7 tons$$

Heat Loss Calculations

$$Q = Q_{TRANS} + Q_{INFIL}$$

where
- Q = total heat loss
- Q_{TRANS} = transmission heat loss
- Q_{INFIL} = infiltration heat loss

$$Q_{TRANS} = UA(T_i - T_o)$$

where
- UA = heat loss coefficient
- T_i = inside air temperature
- T_o = outside air temperature

$$Q_{INFIL} = Q_{SENS} + Q_{LATENT}$$

where
- Q_{SENS} = sensible heat loss
- Q_{LATENT} = latent heat loss

$$Q_{SENS} = V\rho C_p(T_i - T_o)$$

where
- V = volume of air entering building
- ρ = air density
- C_p = specific heat of air

$$Q_{LATENT} = V\rho\ (W_i - W_o)\ h_{fg}$$

where

W_i = inside air humidity ratio
W_o = outside air humidity ratio
h_{fg} = latent heat of vapor at T_i

Simple Equations for Standard Air

$$Q_{SENS} = 0.018 \times V\,(T_i - T_o)$$

$$Q_{LATENT} = 79.5 \times V\,(W_i - W_o)$$

Heat Gain Calculations

$$Q = Q_{TRANS} + Q_{FEN} + Q_{INT}$$

where
Q_{FEN} = fenestration heat gain
Q_{INT} = internal heat gain

$$(Q_{TRANS} / A) = \alpha I_t + h(T_o - T_s) - \varepsilon \delta R$$

where
α = absorptance of surface for solar radiation, no units
I_t = solar radiation incident on surface, Btu/hr ft^2
h_o = heat transfer coefficient, Btu/hr ft^2 °F
T_o = outdoor air temperature, °F
T_s = surface air temperature, °F
ε = emittance of surface, no units
δR = difference between radiation incident on the surface and black body radiation at T_o, Btu/hr ft^2

$$(Q_{TRANS} / A) = h_o(T_{SOL\text{-}AIR} - T_S)$$
$$T_{SOL\text{-}AIR} = T_o + \alpha I_t / h_o - \varepsilon \delta R / h_o$$